Cometaria and the demonstration of Kepler's 1st and 2nd laws

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Introduction

Johannes Kepler (1571 – 1630) empirically derived[1] and published his three famous laws of planetary motion during the first two decades of the 17th century, and they may be stated as follows:

1. Planets move along elliptical orbits, with the Sun at one focus
2. The Sun-planet radius vector sweeps out equal areas in equal intervals of time
3. The cube of a planet's orbital semi-major axis is proportional to the square of its sidereal period.

The initial reactions to Kepler's annunciation of his laws were, apparently, ones of mixed emotion. The first law, for example, was received with basic favour by his immediate contemporaries, but his second and third laws were viewed as being something altogether rather 'suspicious'[2] and, indeed, most astronomers simply choose to ignore them throughout the greater part of the 17th century[3]. Isaac Newton eventually explained the physical principles underlying Kepler's laws in his *Principia Mathematica*, published in 1687, but the practical teaching problem of demonstrating the laws 'in action' to, say, a classroom bound audience of astronomy students, remained steadfastly unsolved. At issue in the model demonstration situation is the point that Kepler's laws are not inherently geometrical, and consequently they cannot be fully described by devices constructed with simple circular gears, levers and/or springs[4]. Starting in the first half of the 18th century, however, a number of 'popularizers' of science did construct mechanical devices that attempted to illustrate the first two of Kepler's laws. Known variously as cometaria, mercuria, and equal-area machines these devices were constructed in order to illustrate motion along an elliptical path and to demonstrate the variation in orbital speed of a planet (or comet) as it moved from perihelion to aphelion - this latter effect being a direct and perhaps more readily observable, in a public demonstration sense, consequence of the second law.

Kepler's problem:

Of the three laws of planetary motion the second is perhaps the most utilitarian in the sense that it provides a means, in principle, of determining the successive locations of a planet, at equal increments of time, as it orbits about the Sun. The second law, however, does not provide a direct means by which the position of a planet can be determined at a specific time. This latter calculation, in fact, falls under the guise of what is known as Kepler's problem, which in modern terms may be expressed as a relationship between the mean anomaly $l = n\,(t - t_0)$ and the eccentric anomaly E. Given that $n = 2\pi / P$, where P is the orbital period, and that t is the time relative to a reference epoch t_0, then the basis of Kepler's problem is to solve the equation

$$l = E - e \sin E \qquad (1)$$

where e is the orbital eccentricity. Once E has been determined [typically via an iterative approach since when $e \neq 0$ there are no closed analytic solutions to (1)] for a given $(t - t_0)$, P, and e, the radial distance r of the planet from the Sun may be calculated from the relationship $r = a\,(1 - e \cos E)$, where a is the orbital semi-major axis. It is often convenient to express the position of a planet in terms of the true anomaly f, which is the displacement angle measured from the Sun to the planet beginning at the perihelion point on the line of apsides. The equation that described the relationship between the eccentric and true anomalies is:

$$\tan(f / 2) = [(1 + e)/(1 - e)]^{1/2} \tan(E / 2) \qquad (2)$$

With the above technical information in place, we may summarize the essential *modus operandi* of a cometarium as being a device for the mechanical solution of Kepler's problem. That is, the uniform input circular motion provided by a hand crank (corresponding to the variation in the mean anomaly) is 'converted' by the cometarium drive system into the non-uniform motion (i.e., the variation of the true anomaly) of a comet marker as it moves along an elliptical track.

A brief history of cometaria and like-machines:

John Theophilus Desaguliers (1683 - 1744) described what was the first mechanical device specifically designed to demonstrate the first two of Kepler's laws - the machine itself being shown[5] to the assembled Fellows of the Royal Society in London on the 8th of March, 1732. Desaguliers device was actually a mercurium[6] and was therefore generally intended to illustrate the elliptical motion of the planet Mercury about the Sun. The front face of Desaguliers mercurium is shown in figure 1a. The figure indicates that the demonstration of Kepler's first law is solved in a mechanically trivial fashion in that the bead P (the planet marker) is simply constrained to move in an elliptical groove. Specifically, by turning the drive handle H the planet marker P is driven around the elliptical path by the actuating arm SO. The circular time dial and the elliptical track are divided into 88 segments, with each segment corresponding to a day's increment of Mercury's motion about the Sun. The eccentricity of the elliptical track, as measured from Desaguliers diagram, is found to be $e \approx 0.67$, which is some three times larger than Mercury's actual orbital eccentricity (e = 0.21). There is probably a practical reason behind the eccentricity adopted by Desaguliers in his mercurium, in the sense that it corresponds closely to the eccentricity of an ellipse with a minor to major axis ratio of ¾ (i.e., e = 0.66). The 'over' eccentricity of the model track, however, was also a favourable design feature, introduced, as Deseguliers put it, "to make the phenomena more sensible"[7] - by which he meant that the variation in the angular velocity of the planet bead P as it moved from perihelion (point L on the diagram) to aphelion (point E on the diagram) was enhanced. That an increased eccentricity results in an enhanced perihelion to aphelion angular velocity ratio may be derived from the standard equation for the variation of orbital velocity in an elliptical orbit. We have that the velocity $V(r)$ at heliocentric distance r is

$$V^2(r) = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \tag{3}$$

where $\mu = GM_{\odot}$, with G being the universal gravitational constant and $M_{\odot}$ being the Sun's mass. At perihelion $r = p = a(1 + e)$ and at aphelion $r = q = a(1 - e)$. So, the angular velocity ratio at these two points is

$$\frac{\omega_p}{\omega_q} = \frac{V(p)/p}{V(q)/q} = \left(\frac{1+e}{1-e}\right)^2 \tag{4}$$

Hence, for Desaguliers model (with $e \approx 0.67$), the ratio ω_p / ω_q is some 11 times larger than that for a model constructed with a true mercurian orbital eccentricity (e = 0.21).

The interior engineering of Desaguliers mercurium is illustrated in figure 1b. The key mechanical innovation introduced by Desaguliers lies in the construction of the drive train which is made of two elliptical pulley wheels. The wheels are constrained so as to roll against each other by a figure-of-eight catgut string wound around their circumference. By the introduction of such a drive train the uniform rotation of the drive handle H about axis G is transformed into the non-uniform rotation of P about the elliptical groove MPL (dashed ellipse in figure 1b) via drive arm SO attached to the axle at S about which the driven elliptical wheel KQVT rotates. We should note at this point that the elliptical pulley wheel drive train introduced by Desaguliers, while innovative, does not actually provide a true demonstration of Kepler's second law[8]. Indeed, only the perihelion to aphelion velocity ratio is correctly reproduced by such drive trains. In consequence, the drive arm SO does not actually sweep out equal areas in equal time, and nor is the correct velocity variation reproduced as a function of true anomaly. When small eccentricity orbits are modeled, however, the error terms are themselves small (see below).

Some 32 years after Desaguliers demonstrated his mercurium to the Royal Society, James Ferguson[9] described a cometarium in his widely read book *Astronomy Explained* (first published in 1764). Millburn and King[10] comment, however, that Ferguson was using a so-called equal-area machine in his public astronomy lectures from as early as 1749. The faceplate and internal drive of the cometarium are shown here in figure 2. The design of Ferguson's cometarium, barring a slight change to the drive handle arrangement (N and H in fig. 2) is identical to that of Desaguliers mercurium. Ferguson comments on the cometarium in his *Astronomy Explained* that, "this curious machine shows the motion of a comet or eccentric body moving round the Sun, describing equal areas in equal times, and may be so contrived as to show a motion for any degree of eccentricity". While Ferguson's comment is true in principle, it is not true in practice, and an inherent design limitation of cometaria is here brought to light: each time an orbit of a different eccentricity is to be demonstrated then (in principle) a new drive train and faceplate would have to be employed. The time dial (the circle to the upper left-hand side of fig. 2) of Ferguson's cometarium is divided into 12 equal divisions, and consequently it was not specifically intended to illustrate the orbit of any particular comet. The orbital eccentricity of the track in Ferguson's cometarium is $e \approx 0.67$, that is, the same as in Desaguliers mercurium. The finely graduated scales of both the time dial and that of the large circular dial surrounding the elliptical track in Feguson's cometarium would have enable a reasonably good calculation of the equation of center to be made. This equation describes the difference between the true and mean anomalies ($f - l$), and is, therefore, a measure of the comets angular deviation from a true uniform circular motion. For small eccentricities the maximum angular deviation will be of order $2e$ radians, and this maximum deviation will occur at times corresponding to ¼ and ¾ of the orbital period P. The equation of center is necessarily zero at both perihelion ($t = 0$) and aphelion ($t = P / 2$).

The machines constructed by Desaguliers and Ferguson have long since been lost, but an extant cometarium built by Benjamin Martin (1704/5? - 1782) circa 1760 forms part of the Collection of Historical Scientific Instruments, at Harvard University (see figures 3a and 3b)[11]. A restored cometarium (figure 4) constructed circa 1750 by John Rix to describe the motion of Halley's Comet is also on display at the Smithsonian Institute in Washington[12]. Two cometaria, one used by Steven Demainbray and the other built by W. and S. Jones, are on display at the Science Museum in London, and form part of the King Charles III Scientific Instruments Collection[13]. Two additional cometaria are held within the collections of the National Museums of Scotland[14]. A 'static' cometarium[15] forms part of the instrument collection at the Museum of the History of Science in Oxford[16].

A true equal-area machine

The elliptical pulley wheel drive train introduced by Desaguliers was by far the most common drive system to be used in cometaria. Indeed, only two designs are known to the authour in which none elliptical pulley wheels or gears were used in the drive train. The first such device was described by James Dean[17], Professor of Mathematics and Natural Philosophy, at the University of Vermont, in 1815. Dean's cometarium used circular gears exclusively. By allowing for a variable distance to be accommodated between the time dial gear center and the center of the gear controlling the non-uniform motion, Dean's cometarium was able to illustrate the effects of changing the modeled orbital eccentricity. Dean's cometarium was also innovative in that the time dial was located at the focus of the elliptical track, allowing for the direct visual measurement of the equation of center. The non-uniform motion in Dean's cometarium was provided for by a variable radius rotation arm constrained to move on a gear whose rotation center was offset from the model's sun focus. W and S. Jones also built a 'variant' device which rather than using elliptical gears used a complex arrangement of an eccentrically mounted circular gear and a variable radius arm to achieve a non-uniform drive motion[18]. Perhaps needless to say, neither of these variant drive trains provided a true demonstration of Kepler's second law. The question that naturally arises at this time, therefore, is what gear or pulley wheel profile, if any, will genuinely provide a true demonstration of Kepler's second law when the orbital eccentricity is non-zero?

When the relationship between the input and the output angular rotation rates of two non-circular gears can be represented by an analytic function, then the required gear profiles may be determined numerically. The general equations that describe the profiles of any coupled gear pair are given by[19]:

$$\varphi_2 = F(\varphi_1) \qquad (5a)$$

$$R_1 = C\, F'(\varphi_1) / [1 + F'(\varphi_1)] \qquad (5b)$$

$$R_2 = C - R_1 = C / [1 + F'(\varphi_1)] \qquad (5c)$$

Where C is the distance between the gear centers, φ_1 and φ_2 are the polar angles (measured in radians) for gear 1 (the drive gear) and gear 2 (the driven gear). Each gear has radii that vary as $R_1(\varphi_1)$ and $R_2(\varphi_2)$. The function $F(\varphi_1)$ describes the relationship between the input and output rotation angles. The function $F'(\varphi_1)$ is the first derivative of $F(\varphi_1)$ with respect to angle φ_1. In the cometarium case we set $\varphi_1 = nt$ (taking without loss of generality $t_0 = 0$). As previously described, $n = 2\pi / P$ is the mean anomaly and t is the time since perihelion passage. In this fashion the variation $0 \le \varphi_1 \le 2\pi$ takes place over the interval $0 \le t \le P$, and since t varies uniformly, so too does φ_1. Next, we impose the identity $\varphi_2 \equiv f$, where f is the true anomaly of the comet at time t. Equations (1), and (2) provide the functional relationship between φ_1 and φ_2, such that

$$\varphi_1 = -\frac{e\sqrt{1-e^2}\,\sin\varphi_2}{1+e\cos\varphi_2} + 2\arctan\left[\left(\frac{1-e}{1+e}\right)^{1/2}\tan\left(\frac{\varphi_2}{2}\right)\right] \qquad (6)$$

where, as before, e is the orbital eccentricity. By specifying the constant C, and the eccentricity of the orbit to be modeled, we may proceed to solve equations (5a) through (5c) for the appropriate gear profiles. Since equation (6) specifies the functional inverse of equation (5a) the profile solution must proceed numerically - but this is a straightforward computational task in the modern era.

Figure 5 shows a series of wheel/gear profiles that will provide exact demonstrations of Kepler's second law for orbital eccentricities of *e* = 0.0, 0.5, 0.75, and 0.95. From figure 5 we see that when the orbital eccentricity is small then the gear profile is essentially that of a circle with radius C/2. In the limit *e* = 0 this result follows exactly from equation (6), since then, $\varphi_2 \equiv \varphi_1$. Indeed, for small eccentricities an eccentrically mounted circular gear will provide a good approximation to the actual profile required to ensure that equal areas are swept out in equal intervals of time. In this sense, the device designed by James Dean[17] would have provided a reasonably good approximation to Kepler's second law for small eccentricities. Indeed, Dean argued (without supporting analysis) that his machine was "capable of representing any degree of eccentricity, from that of Venus [*e* = 0.007] to that of Mercury [*e* = 0.21]"[17]. As the orbital eccentricity increases, however, so the gear profile becomes more 'elliptical' in shape, but we note that the profiles are not true ellipses. Without repeating the details here, it can be shown[8] that the error term (that is the deviation from true Keplerian motion) actually increases as the modeled orbital eccentricity increases in Desagulier-type cometaria. In this manner we find that elliptical pulley wheels or gears, with the same eccentricity as the orbital track being modeled, are never actually the appropriate profiles to use in a cometarium. Intermediate eccentric orbits, on the other hand, may be reasonably modeled through the use of slightly deformed circular gear profiles.

Discussion

The mercurium constructed by Desaguliers was not the first mechanical device to be built with the aim of describing non-uniform celestial motion. Indeed, the astrarium of Giovanni de' Dondi, constructed in the mid-14th Century, employed wheels with slightly elliptical profiles in an attempt to reproduce the motion of mercury[20]. The Moon's movement on the astrarium was also governed by the use of oval gears. Christiaan Huygen's further made a planetarium device in 1682 that employed both centrally offset circular orbital tracks and eccentrically mounted circular gears[21]. Likewise, Ole Rømer in the late 1670s developed a set of conical gears with non-uniformly spaced teeth to drive a planetary marker around a circular orbital track – again, with the idea of describing the non-uniform motion of the planets about the Sun[22]. Although not the first device to describe non-circular motion, Desaguliers mercurium was, however, the first device specifically designed to allow for a demonstration of Kepler's second law.

While a number of instrument makers manufactured various types of cometaria throughout the 18th and early 19th centuries, the basic design changed but very little - the cometaria either having offset circular, or elliptical gear/pulley wheel drive trains. The first serious attempt to move away from the use of an elliptical gear drive train in the simulation of planetary motion was apparently that by Gerhard Schwesinger, chief engineer at the Zeiss optical works in Germany, in the mid-1960s. Schwesinger's patented system[23] employed a set of gears for which the rims could be deformed, by adjustable screws, along two mutually perpendicular diameters. Indeed, Schwesinger's design approach was similar to that described above in the generation of the profiles shown in figure 5. Schwesinger, however, developed a polynomial expression (under a small eccentricity approximation condition) for the profiles.

The cometarium was never seemingly as popular a demonstration device as the orrery or planetarium; devices that were produced in relatively large numbers during the 18th and 19th centuries. Indeed, very few mercuria, equal-area machines and/or cometaria were apparently ever manufactured, and only a handful have survived to the modern era as (known) museum pieces[16]. The cometarium, unlike the orrery or planetarium, was an esoteric device, unsuited in many ways for use in public demonstrations[24] (which is not to say that they were not used by 'popularizers' of science). In addition, since cometaria had only a very limited utility as predictive tools they never became a standard teaching device to be found in every astronomical observatory and/or department of natural philosophy.

Notes and References

1. See, e.g., C. Wilson. *How did Kepler discover his first two laws?* Sci. Am. **226** (3), (1972). pp. 92 - 106. And, J. R. Voelkel. *Johannes Kepler and The New Astronomy*. (Oxford, 1999).
2. J. L. Russell. Kepler's Laws of Planetary Motion: 1609 – 1666. British J. Hist. Sci. **2** (5), (1964), pp.1-24.
3. The third law was problematical since it had neither a geometrical nor a theoretical explanation. Kepler did introduce the idea of a *virtus motrix*, or causal power, emanating from the Sun to 'control' planetary motion but could provide no reasonable explanation for the exact form of the third law [see e.g., P. Barker, and B. R. Goldstein. *Distance and Velocity in Kepler's Astronomy*. Annals of Science, **51**, (1994), pp. 59 - 73]. Isaac Newton eventually explained the third law, in terms of the conservation of angular momentum and universal gravitational attraction.
4. In contrast to Kepler's explanation of planetary motion, the Ptolemaic system was inherently geometrical and could be modeled by mechanical devices constructed of gears and lever arms [see e.g., D. J. de Solla Price. *Gears from the Greeks: the Antikythera mechanism – a calendar computer from ca 80 B.C.* Trans. Am. Phil. Soc. **64** (7), (1974), pp. 5 - 70]. Perhaps the most famous example of a geared, astronomical device showing the motions of the planets according to the Ptolemaic system is the astrarium built by Giovanni de' Dondi between 1348 and 1364 [see S. A. Bedini and F. R. Maddison. *Mechanical Universe: the astrarium of Giovanni De' Dondi*. Trans. Am. Phil. Soc. **56** (5), (1966). pp. 5 – 69].
5. J. T. Desaguliers. Royal Society Record Book for 1732 (RBC 18.245).
6. Desaguliers gave no specific reason for the production of his mercurium, but interestingly he did note that the device could be adapted to describe the orbit of a comet. This latter point being remarkable since it was not until the return of Halley's Comet in 1758 that the periodic nature of at least one comet was established.
7. J. T. Desaguliers. *A course of Experimental Philosophy*. *Volume 1*. (London, 1734).
8. M. Beech. The Mechanics and Origin of Cometaria. *J. Ast. Hist. Heritage*, **5** (2), (2002), pp. 155 - 163.
9. Ferguson was a well-known itinerant lecturer who made his living through public lecture courses [see e.g., J. R. Millburn, and H. C. King. *Wheelwright of the Heavens: the life and work of James Ferguson, FRS.* (London, 1988)].

10. H. C. King, and J. R, Millburn. *Geared to the stars: the evolution of planetariums, orreries and astronomical clocks.* (Toronto, 1978). p. 64.
11. Martin's cometarium is mechanically identical to Desaguliers mercurium, but his device was designed (and sold) to illustrate the motion of Halley's Comet. The elliptical track is accordingly divided into 75 segments, each segment accounting for a year's increment of motion. We note, however, that the eccentricity of Martin's cometarium is the same ($e \approx 0.67$) as Desaguliers original mercurium. D. P. Wheatland in *The Apparatus of Science at Harvard: 1765-1800*. (Boston, 1968) indicates that when purchased in 1766, the Harvard cometarium cost £ 3.12.6 – a not inconsiderable sum of money at that time. I. B. Cohen in *Some Early Tools of American Science* (Russell and Russell, New York, 1950) pp. 54 – 55, further indicates that the cometarium was still being used as a demonstration device some twenty years after its purchase. It was Martin who apparently first coined the name cometarium for Desaguliers-like machines (see, J. R. Millburn's article, Benjamin Martin and the Development of the Orrery, *The British J. Hist. Sci*. **6**. (1973). pp. 378 - 399).
12. The face plate of the cometarium reads "the comet of 1682, J. Rix, Fecit." John Rix was a London mathematical instrument maker who flourished in the mid-18th century [E. G. R. Taylor, *The Mathematical Practitioners of Hanovarian England: 1714 – 1800* (Cambridge, 1966). p. 245]. It has been assumed that the cometarium was fashioned just prior to the1758 predicted return of Halley's Comet. The time dial plate is divided into 75 equal divisions and the eccentricity of the orbital track is $e \approx 0.85$. The actual orbital eccentricity of Halley's Comet is $e = 0.97$.
13. Demainbray's cometarium is described in A. Q. Morton and J. A. Wiess, *Public and Private Science – the King George III Collection* (Oxford, 1993). pp. 160 – 161. Demainbray's cometarium was constructed circa 1755 and appears to be an almost exact copy of Desaguliers mercurium. While the eccentricity of the 'orbital' track is the same in each device ($e \approx 0.67$), Demainbray's cometarium has a time dial divided into 24 equal divisions [the elliptical track is, however, divided into 22 divisions]. A photograph of the cometarium constructed by W. and S. Jones is given in R. Bud and D. J. Warner (Eds.) *Instruments of Science; an historical encyclopedia* (New York, 1998). pp. 125 – 126. See also note 18.
14. One of the cometaria is a vertical demonstration model made circa 1790 by Edinburgh instrument maker John Miller. The cometarium was apparently used for many years as a teaching instrument at the University of Edinburgh. The second cometarium in the National Museums collection was probably built in London by Robert Fidler, circa 1810. This devise was used as a teaching aid in the Department of Physics and Astronomy at the University of St. Andrews. [I am grateful to R. Dempster of the NMS for the information on Fidler's cometarium].
15. By 'static' we mean that the cometarium has no moving wheels or gears. The device at the MHS at Oxford (Inventory number 84080) was designed by John Taylor and built by R. Aidie in 1835. The cometarium has interchangeable plates and arcs that represent the various cometary orbits and these attach to a large circular disc which represents the ecliptic plane. The cometarium in this manner provides a three-dimensional view of a comet's orbit about the Sun. Desaguilers described similar such attachments to a planetarium in a paper read before the Fellows of the Royal

Society on February 21st and 28th of 1733 (Royal Society Record Book for 1733. RBC 18.395). In Desaguliers description wires were bent into various parabolic shapes and attached to the planetarium with the aim of illustrating the "lower part of a comet's orbit". Desaguliers further comments that he has designed the planetarium to show "the orbits of several comets and the periods of three of them". Dutch instrument maker Nicolaas Struyck also described a 'static' cometarium in his *Vervolg van de beschryving der staartsterren* (Amsterdam, 1753). This cometarium illustrated the spatial orientation and extent of some14 cometary orbits [I am grateful to Dr. R. H. van Gent for the bringing Struyck's cometarium to my attention].

16. Brief descriptions of the cometaria referred to in notes 11 through 15, along with their current housings, may be found on the authour's webpage: http://hyperion.cc.uregina.ca/~astro/comet/Index.html.
17. J. Dean, Description of a cometarium, *Mem. Amer. Acad. Arts and Sci*. **3**, (1815). pp. 344 – 345. Webseter's Instrument Makers Database [available on-line from the Adler Planetarium] contains a note that Dean's cometarium was constructed by Aaron Willard, Jr. (1786 – 1864) of Boston, Mass. circa 1830.
18. The cometarium is described in Abraham Rees's, *Cyclopaedia: or Universal Dictionary of Arts, Sciences and Literature*, volume 9, (London, 1819). See also note 10 (p. 208), where it is recorded that the cometarium was priced at £ 5.6.0. in an 1843 catalogue. The exact date of the cometarium's construction is not known, but is believed to be after 1791.
19. N. P. Chirons, and N. Sclater. *Mechanisms and Mechanical Devices Sourcebook*. (2nd Ed. New York, 1966). pp 267 – 270.
20. See note 4 and the reference to S. A. Bedini and F. R. Maddison. p. 15.
21. See W. Pearson, Planetary Machines, in D. Brewster (Ed.), *Edinburgh Encyclopaedia* (Edinburgh, 1830). p. 625. See also note 10, p. 113.
22. See note 10, p. 108.
23. Schwesinger's system is US patent number 03178959. The design for the system was first filed on February 25th, 1963 with registration actually taking place on April 20th, 1965. See also note 10, p. 364.
24. J. R. Millburn describes some of the mechanical problems associated with the use and construction of a modern day cometarium in his article, Demonstrating the motion of comets, *Space Education*, **1** (2), (1981). pp. 55 – 58.

Figure Captions

Figure 1a: Face plate design of Desaguliers mercurium. By turning handle H about axis G, the arm SO drives the planet marker P around the elliptical track at a non-uniform rate. The diagram is from J. T. Desaguliers. *A course of Experimental Philosophy. Volume 1*. (London, 1734).

Figure 1b: Interior design of Desaguliers mercurium. The elliptical pulley wheels (horizontally shaded) are clearly visible in the diagram. The uniform motion of the circular cog at G is converted by the elliptical pulley wheel drive train into non-uniform rotation at axis S. The diagram is from J. T. Desaguliers. *A course of Experimental Philosophy. Volume 1*. (London, 1734).

Figure 2: Ferguson's cometarium design. As in figure 1b, the elliptical pulley wheel drive train is clearly seen on the right hand side of the illustration. Ferguson's cometarium is driven from the side of the machine via a worm gear attached to axle H. This small mechanical innovation over Desaguliers design would have made Ferguson's cometarium a much more practical demonstration device. That is, Ferguson's device is operated from the side rather than from the front. The diagram is from J. Ferguson. *Astronomy Explained upon Sir Isaac Newton's Principles*. 3rd Ed. (London, 1764).

Figure 3a: The face plate of the Harvard cometarium constructed by Benjamin Martin circa 1760. Martin's device is all the more compact for having the circular time dial located within the interior of the elliptical track.

Figure 3b: Interior construction of the Harvard cometarium. The elliptical pulley wheels are clearly visible in the image. The time dial axle is in the foreground of the image and the worm drive responsible for its activation is seen to the left of the central brass plate.

Figure 4: The Smithsonian Institute cometarium constructed by John Rix circa 1750. The face plate shows partial arcs for the orbits of Jupiter and Saturn. The orbit of Uranus is not shown, of course, since it was still some 30 years away from discovery (by W. Herschel in 1781). Rix has added an outer great circle ring to his cometarium to illustrate the motion of the comet against the background stars. Image reproduced courtesy of the Smithsonian Institution, Washington D.C.

Figure 5: True equal-area demonstration profiles. A series of four selected orbital eccentricities (e = 0.0. 0.5, 0.75 and 0.95) are shown. The point (0, 0) indicates the position about which the gear rotates. The constant C has been set equal to unity in the calculations leading to these profiles.

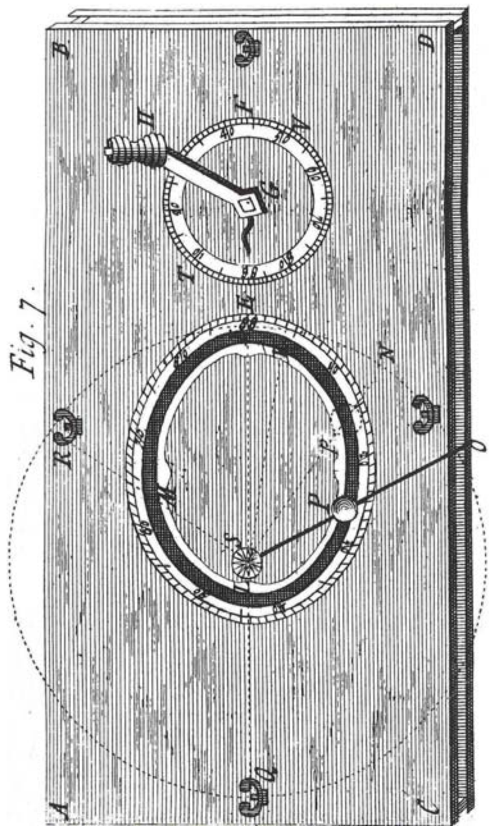


Fig. 1a

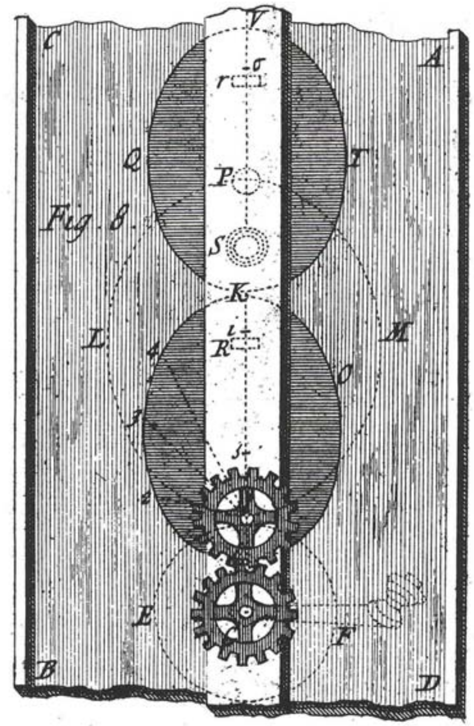


Fig. 1b

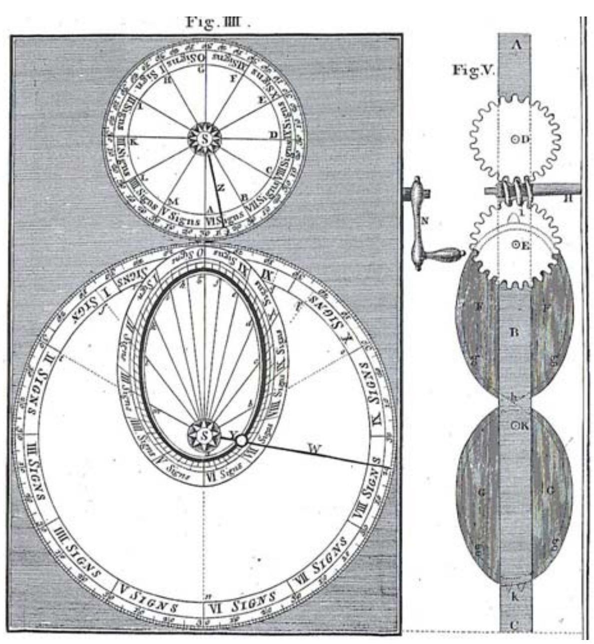
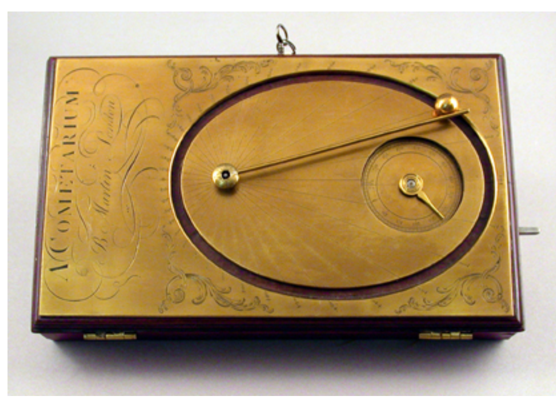
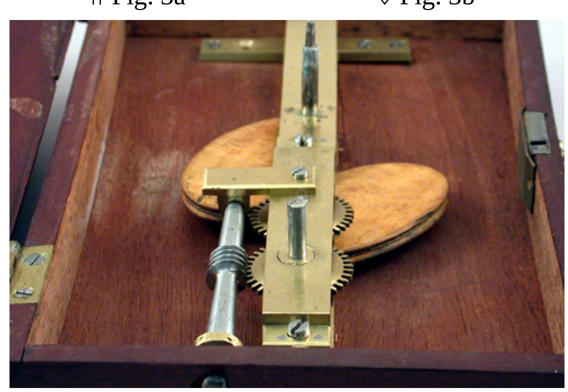


Fig. 2

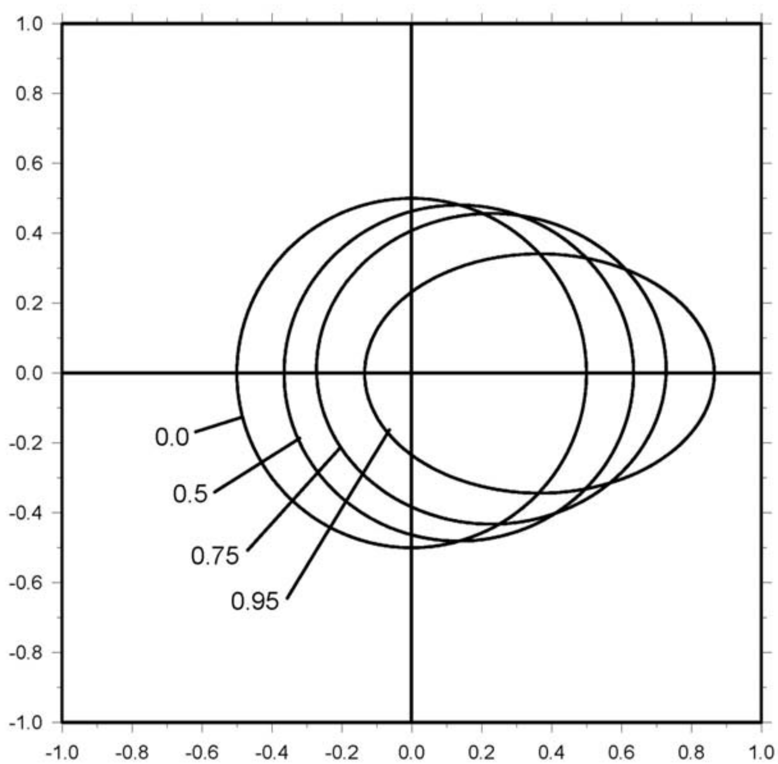


⇑ Fig. 3a ⇓ Fig. 3b



⇑ Fig. 4

Fig. 5



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