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The Mechanics and Origin of Cometaria

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Abstract
The cometarium, literally a mechanical device for describing the orbit of a comet, had its genesis as a machine for illustrating the observable consequences of Kepler's second law of planetary motion. The device that became know as the cometarium was originally constructed by J. T. Desaguliers in 1732 to demonstrate, in a sensible fashion, the perihelion to aphelion change in velocity of the planet Mercury. It was only with the imminent, first predicted, return of Halley's comet in 1758 that the name cometarium was coined, and subsequent devices so named. Most early cometaria used a pair of elliptical formers joined via a figure-of-eight cord to translate uniform drive motion into the non-uniform motion of an object moving along an elliptical track. It is shown in a series of calculations, however, that two elliptical former cometaria do not actually provide a correct demonstration of Keplerian velocity variations and nor do they actually demonstrate Keplers' second law of planetary motion.

Key words: Comets; cometary orbits; Orreries.

"Now we know the sharply veering ways of comets"

Edmund Halley, "Ode to Newton" 1686

1. INTRODUCTION

On Thursday the 8th of March 1732, John Theophilus Desaguliers demonstrated to the assembled Fellows of the Royal Society an instrument "to show the different velocities of a planet or comet in its motion round the Sun" (Desaguliers, 1732). The device displayed by Desaguliers was like no other machine then in existence and it had been especially designed for the purpose of 'illustrating' Kepler's second law, which requires that the planet-Sun vector sweeps out equal areas in equal time. Desaguliers did not apply a name to his new device, but it, and subsequent devices like it have at various times been called equal-area machines, mercuria and cometaria.

From an engineering perspective the problem with Kepler's second law is that it cannot, in fact, be explained or easily illustrated by mechanical means. As Isaac Newton showed in his *Principia*, fist published in 1687, Kepler's laws are a manifestation of the principles of universal gravitational attraction and the conservation laws of energy and angular momentum. The point being that none of these deep physical concepts can be exactly described with the aid of mechanical gears, springs and/or linkages. Desaguliers machine, therefore, was designed to illustrate (or mimic) the 'potentially observable' consequences of the second law, which most noticeably for the observer would be a marked decrease in the angular velocity of the model planet (or comet) as it moved from perihelion to aphelion. Desaguliers' machine was in this latter context neither a fully predictive nor an explanatory device. This situation can be offered in contrast to the other 'planetary' machines that had been constructed at that time. The telluria and lunaria first constructed circa 1715, were used, for example, not only to demonstrate the scale of the Solar System (that is relative orbital size and planetary motion), but also to explain such effects as eclipses, phases of the Moon and the reasons why the Earth experiences seasons (King and Millburn, 1978).

In the sections that follow we shall describe in some detail the workings and construction of Desaguliers' machine, and we shall then briefly outline a few of the mechanical developments introduced by other instrument makers.

2. A DEVICE AHEAD OF ITS TIME

While, as stated above, Desaguliers offered no particular name for the machine he demonstrated to the Royal Society, it may be reasonably described as a mercurium – that is, a device for illustrating the orbital motion of Mercury about the Sun. The association with Mercury arises not because of the orbital eccentricity being modeled but because the time scale of the device was divided into 88 equal intervals, and this as Desaguliers (1732) wrote was "the number of days of Mercury's revolution". No motivation for adopting the orbit of Mercury is given by Desaguliers, but in 1732 it was certainly the planet with the largest known eccentricity and therefore also the planet with the greatest variation in its velocity upon moving from perihelion to aphelion. Recall that adherence to Kepler's second law requires that the velocity at perihelion be greater than that at aphelion. In addition we note that Mercury was due to undergo a solar transit on November 11, 1736, and was consequently an object of up-coming interest with respect to the determination of the astronomical unit (see e.g., Hughes, 2001).

While it seems clear that Desaguliers had the planet Mercury in mind when he constructed his device, he also suggested it could describe the orbital motion of a comet. This latter possibility is, in fact, historically rather interesting and indeed a potentially controversial statement for Desaguliers to have made. When Desaguliers constructed his machine circa 1732 it was neither clear how big comets actually were nor what they were made of. Nor, indeed, was it absolutely clear that comets orbited the Sun along elliptical orbits – that is, that comets were periodic. Certainly Newton and Halley had argued that some comets moved along elliptical orbits, but it was not until 1758, with the first predicted return of comet 1P/Halley, that the periodic nature (of at least one comet) was demonstrated. With respect to the nature of comets Desaguliers noted in his *A course of Experimental Philosophy* (Desaguliers, 1734:409) that "comets are a sort of excentrick planets, which move in very long ellipses." To which he continues "whose periodical revolutions take up such a long space of time that the same man has never yet seen the same comet twice." Later in his text Desaguliers notes that "the comets are reckon'd not to be less than the Moon, nor much bigger than Venus." Desaguliers views on the nature of comets in his *Course* generally run parallel with those earlier espoused by Newton in his *Principia* (see e.g., Schechner-Genuth, 1997). It was presumably Desaguliers unequivocal belief in the correctness of Kepler's laws of planetary motion and Newtonian gravitation theory[1] that lead him to suggest that his mercurium could also describe the orbit of a comet some 26 years before the fact was demonstrated observationally.

3. MERCURIUM MECHANICS

At best it is only the first two of Kepler's three laws of planetary motion that can be illustrated by mechanical means. Kepler's third law, which relates the square of the orbital period to the semi-major axis of the orbit cubed, has no mechanical analog and is a result that stands by empirical measurement and Newtonian gravitational theory alone. Kepler's first law of planetary motion, on the other hand, which states that the planets move along elliptical orbits with the Sun at one focus, can be easily accommodated in any mechanical device by simply making a planet-marker move along an elliptical track. We note, however, that rather than fully accommodate the first law most early instrument makers simply had planet markers move along circular tracks with the Sun offset from the center. A nice example of such an 'offset Sun' construction can be seen in the impressive Eisinga planetarium, constructed circa 1780 (see figure 6 of Mulder de Ridder, 2002).

Figure 1 shows the front face of Desaguliers' mercurium. The Sun is located at focal point, S, and the planet-marker, P, is carried around in an elliptical track by drive arm SPO when the handle GH is turned. The circumference of the elliptical track is divided into 88 segments, each segment representing a day's worth of Mercury's orbital motion. The eccentricity of the elliptical track is 0.67, some three times larger than Mercury's actual orbital eccentricity (e = 0.21). This exaggeration of the orbital eccentricity was quite deliberate and introduced by Desaguliers (1732) "to make the phenomena the more sensible". And indeed, this is a reasonable enough step given that the device was designed solely to 'illustrate' the effect of velocity changes at perihelion and aphelion.

The demonstration of Kepler's second law, which requires that the planet-Sun arm sweeps out equal areas in equal time, is a far more complex mechanical demonstration than that for the first law. The inherent engineering difficulty is that Kepler's second law requires that a non-constant orbital velocity be accommodated. Desaguliers was fully aware of this point and consequently developed an elliptical pulley system to deliver a non-constant rotation rate to the planet drive arm SPO (figure 1).

Figure 2 shows the interior of Desaguliers' mercurium. The two elliptical formers are linked via a figure-of-eight cord and the constant rotation rate of the drive ellipse about focus I is transformed into the non-constant motion of the driven ellipse about S. The non-constant motion about focus S is directly transmitted to the drive arm SPO and the planet/comet marker is correspondingly driven about the elliptical track with variable velocity (see, however, Appendix A2 for a mathematical description of what actually transpires).

Desaguliers (1732) explained in his account to the Royal Society that his instrument was constructed to "show the different velocities of a planet or comet in its motion around the Sun, …, describing … areas proportionable to the times, [with] the velocities of the revolving body being reciprocally as the distance from the central body." In his secondary statement about velocities, Desaguliers is referring to the 'inverse distance' form of Kepler's second law. Indeed, Kepler initially presented his second law in two forms. One form expounded the law of areas, while the other stated that the velocity (or as Kepler called it the 'delay' – see e.g., Martens, 2000; and Russell, 1964) of a planet varies inversely with heliocentric distance. Kepler initially believed that these two forms were equivalent statements of what we now call the second law, but in the general case they are not the same. For small values of the eccentricity, however, the inverse distance law is approximately true. We can see that this is so by expanding the velocity, V, in to its radial and angular velocity components (see e.g., Szebehely, 1989 and figure 3) such that $V^2 = (dr/dt)^2 + (r d\nu/dt)^2$. We may now argue that since the radial component of the velocity, dr/dt, will become small as the eccentricity approaches zero, so $V \approx r\, d\nu/dt$ in the small eccentricity limit. Further, given the equal area rule we have $r^2 (d\nu/dt) =$ constant (see e.g., Szebehely, 1989) and, hence, by substitution we find that V is inversely proportional to the radius, r. The point of this argument, of course, is that the inverse distance 'law' is only approximately true under the small eccentricity condition and is not as such equivalent to the conservation of angular momentum argument. Russell (1964) notes, however, that by the time that Kepler published his *Epitome Astronomiae Copernicanae* (in three parts between 1618 to 1621) he had realized that the inverse distance law only held true for small eccentricities.

Even though Kepler provided a clear statement of his second law, it was often ignored or rejected by his contemporaries for less accurate but more easily applied approximations. Seth Ward (1617 – 1689), Savilian Professor of Astronomy at Oxford, for example, calculated planetary positions by considering the ‘empty’ focus of the orbit to be an equant point – an approximation that holds, in fact, to good accuracy for small eccentricities (see e.g., Evans, 1998). From an engineering perspective it would have been simpler to build a model under Ward’s scheme, but it, of course is not at all consistent with Kepler’s laws which make no reference to the second, empty focus. While his calculating methods are now known to be suspect, Gunther (1967:80) argues that it is to Ward that we should attribute the idea that comets actually move about the Sun on closed orbits. All this being said, however, by choosing elliptical formers to modulate the rotation rate of the drive arm (SPO in figure 2) in his mercurium Desaguliers was at least able to demonstrate a marked variation in the ‘bead’ velocity between perihelion and aphelion. Interestingly, the elliptical formers only provide the correct perihelion to aphelion velocity ratio (Millburn, 1981): $V_{per} / V_{aph} = (1 + e)/(1 - e) = Q / q$, where e is the orbital eccentricity, Q is aphelion distance and q is the perihelion distance (see Appendix A2). While the ratio of the velocities is correctly reproduced with elliptical formers the actual perihelion and aphelion velocities are incorrectly modeled with respect to Keplerian motion. The perihelion and aphelion velocities are both, in fact, too small with respect to Keplerian motion in an elliptical former cometarium by a factor $\sqrt{(1 - e^2)}$.

4. DISCUSSION

In the written account of Desaguliers demonstration to the Royal Society, no explanation is offered as to genesis and development of the mercurium. It is clear, however, that Desaguliers was experimenting with the design and construction of instruments for representing planetary motion in the early 1730s. His new and innovative planetarium, described in 1733, for example, was built explicitly for use in his lecture courses and with “the desire of giving a true notion of the celestial phenomena in the plainest and most expeditious mannor” (Desaguliers, 1733). While great attention was directed towards accurately representing the relative size of each planet’s orbit, the relative orbital periods, the relative sizes of the planets themselves and their orbital inclinations, no attempt was made to illustrate Keplerian motion in the planetarium. One presumes that the Keplerian aspect was ignored in the planetarium model because of the complex mechanical requirements that it would call into effect. And indeed, it is only Mercury that has an appreciable orbital eccentricity, and the other planetary orbits are well approximated by circles. After Mercury, Mars has the next most eccentric orbit (of the planets known circa 1733), but with an eccentricity e = 0.093 it is some 2.2 times smaller than that of Mercury’s orbit. Interestingly, Desaguliers (1733) comments that his planetarium could be fitted with bent wires to illustrate the “parabolic figure … of a comet’s orbit” and that specifically the planetarium had been designed to show “the orbits of several comets and the periods of three of them.” The three periodic comets that were modeled presumably correspond to those studied by Halley (e.g., the comets of 1680, 1661 and 1682, although it should be noted that Halley’s ‘demonstration’ of the periodic nature of the comets of 1680 and 1661 was incorrect). Desaguliers planetarium has long been lost, but was apparently last on display, circa 1813, in the Royal Military Academy in London (King and Millburn, 1978:170). It seems worth commenting at this stage that the problem of ‘sensibly’ demonstrating cometary motion by mechanical means still exists to this day. In recent times, however, Hughes (1985) has suggested the construction of a large-scale model of comet Halley's orbit around which 76 posts could be placed at separations corresponding to the distance traveled by the comet in successive one-year time intervals. In such a construction the 'crowding' of posts near aphelion would illustrates the slow motion of the comet when far from the Sun. Tattersfield (1984) has also described, in recent times, the construction of a static, but detailed three-dimensional card model for the orbit of Halley's comet. Of course, in the most recent era, computers have been very successfully utilized in the study and visualization of complex dynamical behaviour, but such demonstrations are clearly not 'mechanical'.

When describing the mercurium in his *A Course of Experimental Philosophy* (Vol. 1 published 1734; p. 446), Desaguliers comments that it is a "machine to show mechanically, how planets and comets, by a Ray drawn from the Sun, describe areas proportionable to the time." In this later work, we note, Desaguliers has dropped the 'inverse distance' law for the velocity variation. We also note here, however, that as with the representation of orbital angular velocities, the arrangement of elliptical formers used by Desaguliers does not actual provide an equal area demonstration (see Appendix A2).

The term cometarium was apparently first used by Benjamin Martin in the early 1740s. Specifically in his *Course of Lectures,* Martin indicates that the Copernican model of the Solar System will be explained by the mechanical orrery and cometarium (Millburn, 1973). Presumably prompted by the imminent return of Halley's comet (in 1758/9), Martin further built and marketed a cometarium with his book *The Theory of Comets Illustrated*, published in 1757 (Taub, 1998; see also Stephenson *et al.,* 2000 and Wheatland, 1968 for woodcut illustrations of Martin's cometaria). James Ferguson, who actually sold his instrument making business to Benjamin Martin in 1757 (Rothman, 2000), in his *Astronomy Explained upon Sir Isaac Newton's Principles*, published in 1756, also described in detail the workings of a cometarium and equal-area machine (Henderson, 1867). We note that the dial plate of Ferguson's cometarium is divided into 12 divisions, rather than Desaguliers 88, and was, therefore, not specifically intended to illustrate the orbit of Halley's comet or Mercury. We also note that Ferguson's 'cometarium' is not an exact mechanical copy of Desaguliers original. Specifically a 'worm gear' is used to drive the elliptical pulleys and the time display dial in Ferguson's machine. This innovation would have been useful during a lecture since it would enable the device to be 'hand cranked' from the side of the device as opposed to the front face as in Desaguliers construction (see figure 1). Henderson (1867) adds an interesting foot note to his commentary on Ferguson's cometaria and relates that "cometariums constructed on this plan [using elliptical formers], and sufficiently large for the lecture room … cost of about £2 10s.; when made with eccentric wheels (instead of pulleys and cat-gut strings), the price may rise to £4." In similar fashion to Ferguson, it appears that Stephen Demainbray also used a cometarium device in his public lectures on planets and comets from about 1755 onwards. It does not appear, however, that Demainbray actually used the term 'cometarium' to describe his model (Morton and Wess, 1995). Demainbray's extensive collection of scientific instruments (including his cometarium) now form part of the King George III Collection of scientific instruments at The Science Museum in London. The elliptical track in Demainbray's cometarium is divided into 22 segments, so, again it was not intended to specifically illustrate the motion of Halley's comet. The division into 22 segments is possibly a simple one-quarter reduction of the 88 day mercurium dial used by Desaguliers.

During the later part of the 18th century it appears that a number of scientific instrument makers were constructing various forms of cometaria (Olson and Pasachoff, 1998). The early designs, as employed by Desaguliers, Martin, Ferguson and Demainbray, used elliptical formers connected via a figure of eight cord. This arrangement was not entirely satisfactory, however, as the cord was prone to slip from the formers and the systems were apparently tricky to re-set. Interestingly, the choice of connecting cord material was one of the main problems encountered in the construction of a modern day version of a Desaguliers'-type cometarium (Millburn, 1981). To circumvent cord slippage, some instrument makers employed meshed elliptical gears in their cometaria. An extant example of a geared cometarium built for classroom use is that held in the collection of the Royal Museum of Scotland, Edinburgh (Holbrook, 1992). The machine was built by scientific instrument maker John Miller (c. 1771 – 1804) [see Bryden, 1972] in the late 18th century for the Department of Natural Philosophy, University of Edinburgh. While the application of meshed gears solved the cord slippage problem it was a work intensive (hence expensive) and technically difficult way of 'correcting' a somewhat trivial problem associated with the operation of the original machines.

Rather than cut expensive elliptical gears William and Samuel Jones manufactured a cometarium which used an eccentrically mounted circular gear with a sliding roller system to ensure constant mesh with the actuating pinion. A cometarium by W. and S. Jones is on display in The Science Museum, London (see also the illustration in Olson and Pasachoff, 1998:47). King and Millburn (1978:208) note that in their 1855 catalogue the Jones's list the cometarium as costing £5. 6s.0d. Perhaps the least complicated design of a meshed, circular gear cometarium is that described by Dean (1815). In Dean's model the varying rate of cometary motion is produced by allowing a variable radius drive arm control the Sun-comet position arm. Interestingly, and unlike all of the other cometaria described in this article, Dean's cometaria could be set to accommodate a range of eccentricities. Sadly, no extant mechanical realization of Dean's cometatium is known at the present time.

Not quite one year on from the time that he presented his mercurium, Desaguliers (1733) described to the assembled Fellows of the Royal Society the workings of his new planetarium. By way of introducing his device Desaguliers argued that "machines and movements for representing the motions and appearances of heavenly bodies have been justly esteemed in all ages." While even to the modern day this statement can be readily defended, the golden age of mechanical orreries, planetaria and cometaria, as valued scientific teaching tools, was relatively short lived. Indeed just one hundred years on from the inaugural description of Desaguliers mercurium, we find Sir John Herschel in his *A Treatise on Astronomy* describing such instruments as "those very childish toys"[2]. But, childish toys or not, we prefer to remember the cometarium and allied machines, out dated as they presently may be, in terms of the lines[3]

"When lately Jove the Orrery survey'd
He smiling thus to Gods in Council said
How shall we stint presuming Mortals Pow'r?"

5. NOTES

1. Deasaguliers was both a good friend of Sir Isaac Newton and an important promoter of his work. And, reciprocally it was essentially upon Newton's suggestion that Desaguliers became 'curator' of experiments at the Royal Society (Hall, 1970). Finding much more than 'new physics' within Newton's *Principia*, Desaguliers suggested that Newton's ideas should be applied to all fields of human endeavour (including politics). In his *The Newtonian System, an allegorical poem*, published in 1728 Desaguliers made his feelings towards Newton's greatness very clear:

   "Newton the unparallel'd, whose name
   No Time will wear out of the Book of Fame,
   Coelestiall Science has promoted more,
   Than all the Sages that have shone before."

2. Sir John Herschel was a close friend of Charles Babbage, and at the time that he would have been writing *A Treatise on Astronomy* (i.e., circa 1832), the first, and only successfully constructed and fully functional piece of Difference Engine No. 1 was delivered to Babbage (Swade, 2000). The Difference and Analytic engines of Babbage were certainly no toys, but were mechanical devices that pushed the then available technology to its limits. Indeed, only small test segments of the various machines designed by Babbage were ever produced in his lifetime. In the light of these events, and given Herschel's close involvement with the political problems associated with the funding of Babbages machines, his comments concerning Orreries are more understandable.

3. Gunther (1967:269) provides the poem, the first three lines of which are given in the text, to the Orrery by "J. H a fellow of the RS" written in 1719. The poem ends by suggesting that John Rowley should be "transplanted" to heaven and made a "star". It was Rowley who made a lunarium (a Sun, Earth and Moon system) for Charles Boyle, 4th Earl of Cork and Orrery, circa 1713 (King and Millburn, 1978:154) and by this act the name Orrery became synonymous with celestial mechanical models in general.

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APPENDIX A

A.1: Orbital motion

The ratio of the variable rate of change, (dν/dt), of the true anomaly ν of a planet in an elliptical orbit of eccentricity *e* to the constant mean angular motion, n, of an object in a circular orbit having the same period of revolution as the planet is

$$\frac{(\mathrm{d}\nu / \mathrm{dt})}{\mathrm{n}} = \frac{(1+e\cos\nu)^2}{(1-e^2)^{3/2}} \qquad (1)$$

where n = 2π / P, with P being the orbital period. Equation (1) is established by a straightforward application of the conservation of angular momentum.

A.2: Elliptical gearing

In a Desaguliers'-type mercurium one of the two identical elliptical gears rotates at a constant angular rate ω_1 about its focal point F_D (see figure 3). The second gear is thereby caused to be driven at a variable rotation rate ω_2 = dν/dt about F_P. The variable rotation ω_2 is transmitted to a planet marker, moving in an elliptical groove, by a drive arm attached to F_P. The ratio of the angular rates is given by the equation

$$\frac{\omega_2}{\omega_1} = \frac{\mathrm{Z}^2 + 1 + \left(\mathrm{Z}^2 - 1\right)\cos(\nu)}{2\mathrm{Z}} \qquad (2)$$

Where ν is the angle through which the driven elliptical gear rotates about F_P and where Z = (1 + e)/(1 – e) is the ratio of the maximum and minimum distances from the focus. If the mercurium is to correctly model the orbital motion of a planet (or comet) then equations (1) and (2) will have to be identical. However, we find an error term f(ν) in the mercurium, such that

$$\mathrm{f}(\nu) = \frac{\omega_2/\omega_1}{(\mathrm{d}\nu/\mathrm{dt})/\mathrm{n}} = \sqrt{(1-e^2)}\,\frac{1+2e\cos\nu+e^2}{(1+e\cos\nu)^2} \qquad (3)$$

we can readily see from equation (3) that the ratio f(0) / f(π) is unity, which indicates that the mercurium provides the correct perihelion (ν = 0) to aphelion (ν = π) angular velocity ratio. The actual perihelion and aphelion velocities provided by the mercurium are smaller than the Keplerian orbital velocities, however, by the factor $\sqrt{(1-e^2)}$. In addition, we see from equation (3) that a Desagulier's-type mercurium provides the correct orbital angular velocity just four times per orbit at the positions corresponding to f(ν) = 1. The variation of f(ν) against ν, as given in equation (3), for various values of eccentricity is shown in figure 4.

In addition to the Desaguliers'-type cometarium having a velocity variation error term f(ν), it also has an area swept out per unit time error term such that $(dA/dt)_{cometarium}$ / $(dA/dt)_{Kepler}$ = f(ν), where A is the area swept out, f(ν) is again given by equation (3), and where $(dA/dt)_{Kepler}$ is the constant appropriate to Kepler's second law.

Figure Captions

Figure 1. Front piece of Desaguliers mercurium. Photograph courtesy of the Royal Society.

Figure 2. Interior view of Desaguliers mercurium. Photograph courtesy of the Royal Society.

Figure 3. Elliptical former arrangement for Deasguliers'-type mercurium. The drive and driven formers are constructed to have the same semi-major axis (*a*) and eccentricity (*e*). The drive former rotates at constant angular velocity ω_1 about F_D. The resultant motion of the driven former about F_P is the variable angular velocity ω_2. A drive arm is attached to the driven focus support F_P and this carries a planet marker or bead around an elliptical track (shown as a dashed ellipse in the figure). The velocity V of the bead in its track about F_P can be expressed in terms of the radial and angular velocity components dr/dt and r(dν/dt) respectively.

Figure 4: Elliptical former error term f(ν) plotted against true anomaly ν. Loci of f(ν) are shown for a selection eccentricities. Mercury has an orbit of eccentricity 0.21, while the eccentricity of Desaguliers' mercurium was 0.67. The loci for e = 0 (circular orbit) and 0.5 are for illustrative comparisons.

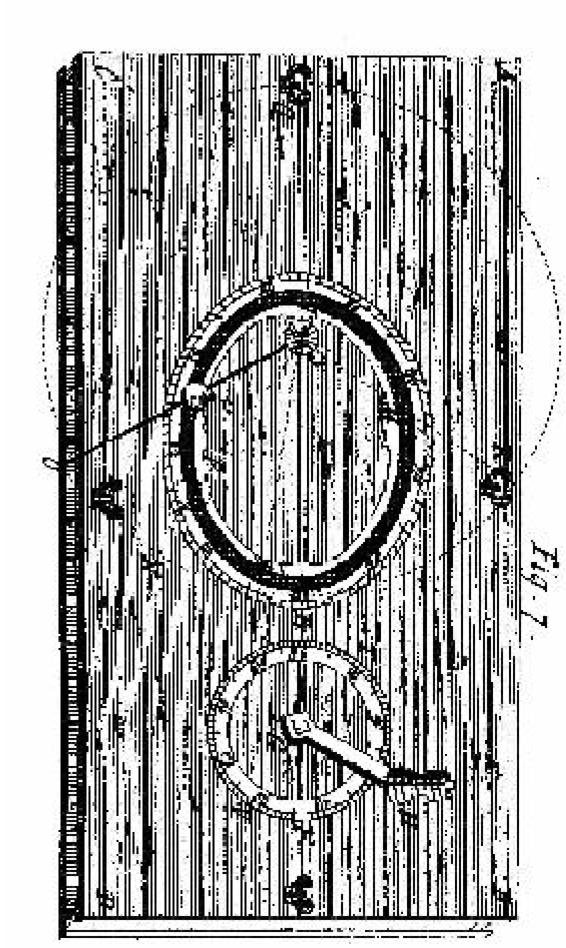


Figure 1

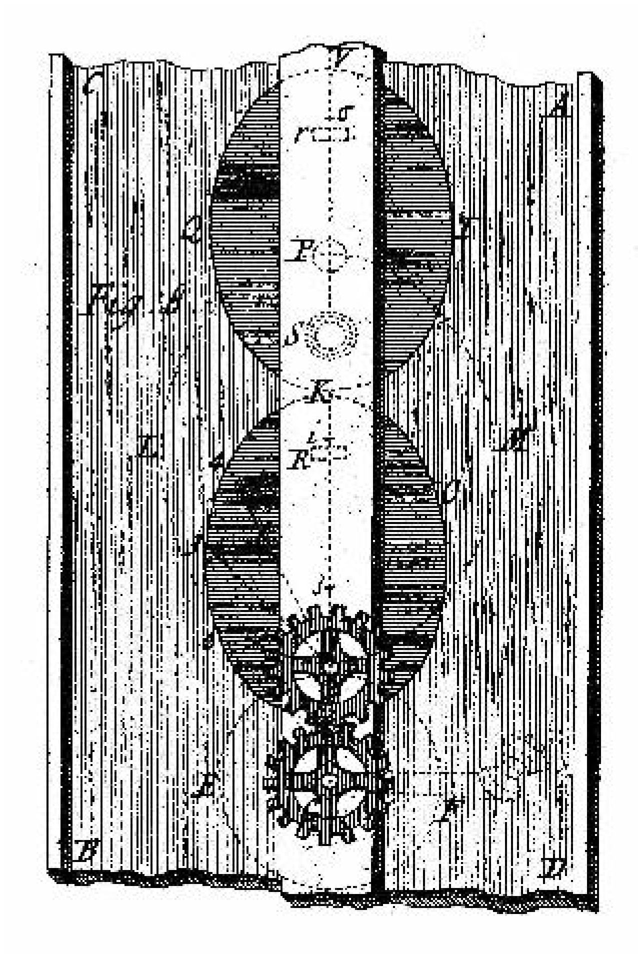


Figure 2

Figure 3:

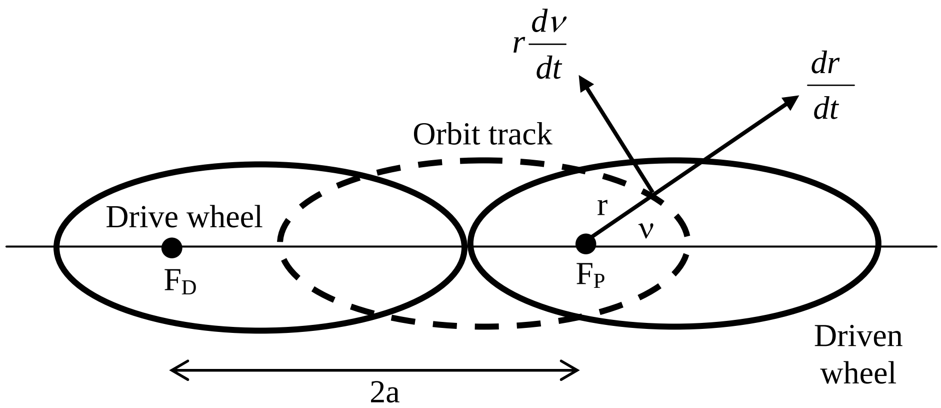
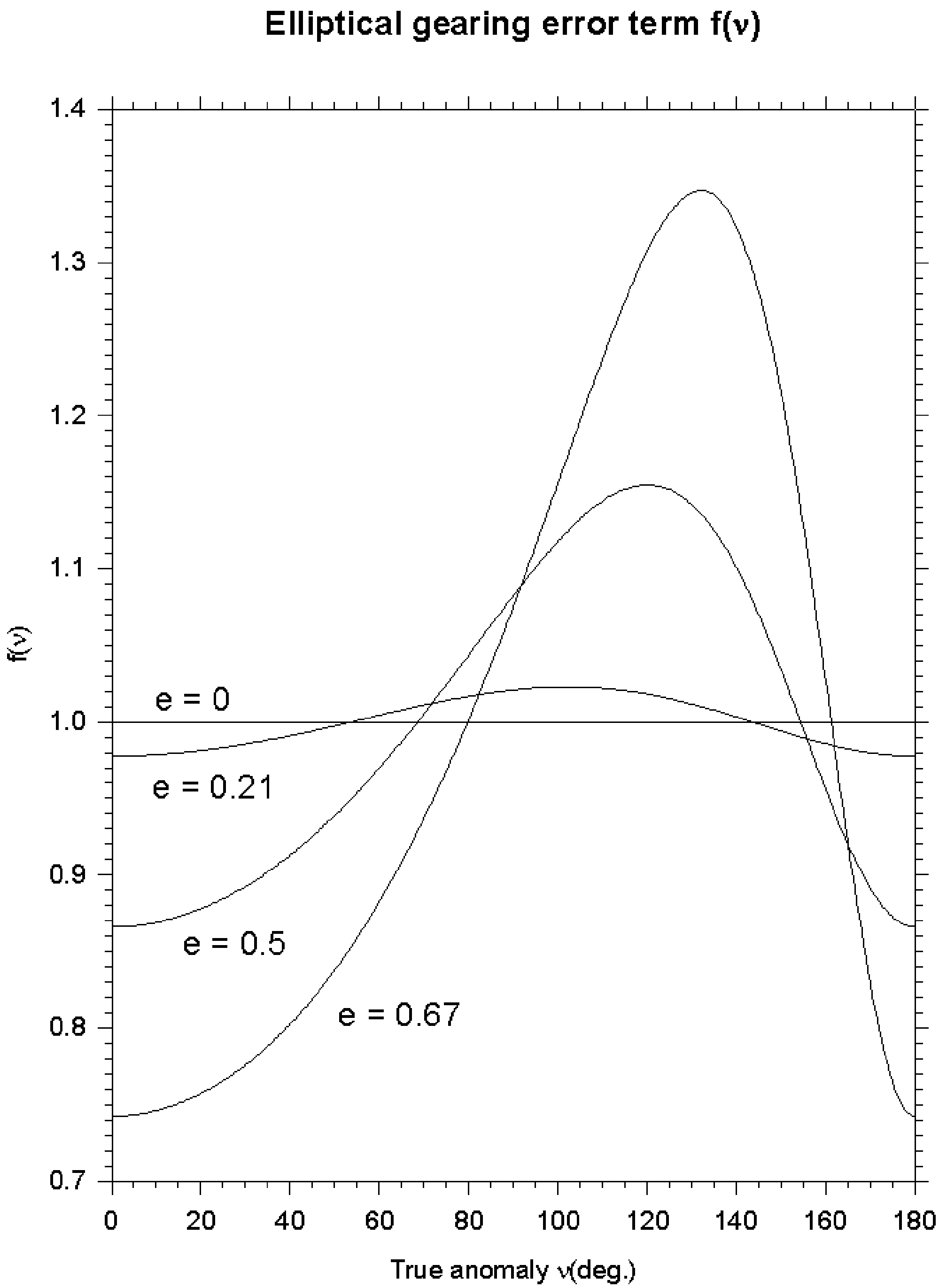


Figure 4.


Elliptical gearing error term f(ν)
f(ν)
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
e = 0
e = 0.21
e = 0.5
e = 0.67
0
20
40
60
80
100
120
140
160
180
True anomaly ν(deg.)