In the shadow of Aristarchus and the lunar eclipse of February 20th, 2008

By Martin Beech

For all his fame and renown Aristachus of Samos is a rather mysterious figure within the history of astronomy. He was born towards the beginning of the 3rd Century B.C. on the Greek Island of Samos, and died circa 230B.C. For his contributions to astronomy Aristarchus is accredited with the invention of the hollowed-out bowl-shaped *scaphe* sundial, and he is remember for his suggestion that the Earth might actually move around the Sun with the sphere of the stars being fixed, or unmoved. Aristarchus also outlined a method by which, when the Moon is at either 1st or 3rd quarter illumination, the distance of the Sun from the Earth can be determined in units of the Earth-Moon separation. The relative distance he derived, however, was an underestimated of the true value by a factor of about twenty, but this is probably because he guessed rather than attempted to measure the angle required to complete the analysis. In contrast, in the one surviving text by Aristarchus, *On the sizes and distances of the Sun and Moon*, that has come down to us through history he outlines the correct reasons for the phases of the Moon and he also describes a potentially accurate method by which the distance to the Moon can be determined in units of the Earth's radius $R_{\oplus}$. In the latter case, the method he outlines is partly based upon the measurement of the cross-section size of the Earth's shadow at the Moon's orbit during the time of a total lunar eclipse. Aristarchus assumed the shadow width to be twice that of the Moon's angular diameter – a diameter corresponding, therefore, to 1 degree on the sky. This is not a particularly good estimate (although it is close to being correct). Claudius Ptolemy working several hundred years after the death

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of Aristarchus, circa 100 A. D., in Book V of his *Magna Syntaxis* gives a more reasonable estimate of 13/5 times the Moon's diameter – corresponding to an angle of 1.3 degrees for the width of the Earth's shadow. In the work of Aristarchus we indeed find both the great strength and the overpowering weakness of classical Greek astronomy. It wasn't the thinking or the methodology that was in any way wrong; it was the poor means that they at their disposal with which to measure very small angles on the sky with any degree of precision.

I have seen many eclipses, both lunar and solar, over the years and was not immediately excited by the February 20th eclipse prediction when I first read about in the fall of 2007. By great and good chance, however, I was reminded of Aristarchus's study concerning the lunar distance while reading through James Evans wonderful book *The History and Practice of Ancient Astronomy* (Evans, 1998). Here, it immediately struck me, was a chance to re-live a small part of astronomical history. Having previously measured, a number of years ago now, the Earth's radius according to the methods of Eratosthenes (Beech, 2005) it was time to delve into the deeper realm of the incorruptible heavens.

The observing conditions at Regina, in spite of all expectation, were perfect for the total lunar eclipse on February 20th, 2008. The days before had been blustery and overcast, but on the night of the eclipse the skies opened-up to reveal a glorious star-studded, moonlit vista and a cold, crisp observing session began (figure 1).

Figure 1. A 3½ hour-long exposure, starting at 19:30 CST, of the lunar eclipse unfolding. The picture was taken with a fish-eye lens on a Nikon camera body using standard 35mm colour print film. A close examination of the image reveals trails for Sirius and Mars.

Figure 2 shows the geometry that I am going to apply in this analysis. The center of the Earth is at position O, and the Moon's orbit (assumed circular – of course) is shown as the dashed track. Aristarchus noted that during a solar eclipse the Moon would (typically) just cover the Sun's disk and this, he argued, indicated that the Sun and Moon must have nearly same angular diameter on the sky. Archimedes of Syracuse (827 – 212 B.C) in his *Sand Reckoner* further tells us that Aristarchus discovered that the Sun has an apparent angular size of 1/720th part of the zodiacal circle, which is an angle of 0.5 degrees - a remarkably good angle determination in this case. The solar eclipse condition is shown on the right-hand side of the diagram, and this allows us, as it did Aristarchus beforehand, to take angle $\varphi = 0.5^o$. Constructing lines parallel to the full shadow rays associated with the solar eclipse condition, and extending them so that they are tangential to the Earth allows us to construct the Earth's full shadow zone (the Earth's umbra) which reaches a point at P on the far left of the diagram. To close the analysis what we require from the lunar eclipse observations is an estimate of the angular width of the Earth's shadow at the Moon's orbital distance. This observation will provide us with an estimate of the angle α. If we take the Moon's orbital radius to be $D_{☾}$ then, without going through all the trigonometric details, we have

$$D_{☾} / R_{\oplus} = [2 - (2 + \varphi) / (1 + \varphi / \alpha)] / \varphi \qquad (1)$$

where we have used the small angle approximation with φ and α being expressed in radians. Clearly equation (1) and the use of radians was not the approach taken by Aristarchus, who would have used a more geometrical method of analysis, but we will allow ourselves some degree of modern convenience. We already have one number at our disposal: $\varphi = 0.5^o = 0.0087$ radians. To estimate α, we need to look at the Moon during the partial eclipse phase. Figure 3, illustrates the method used to determine the radius of the Earth's shadow at the Moon's orbit.

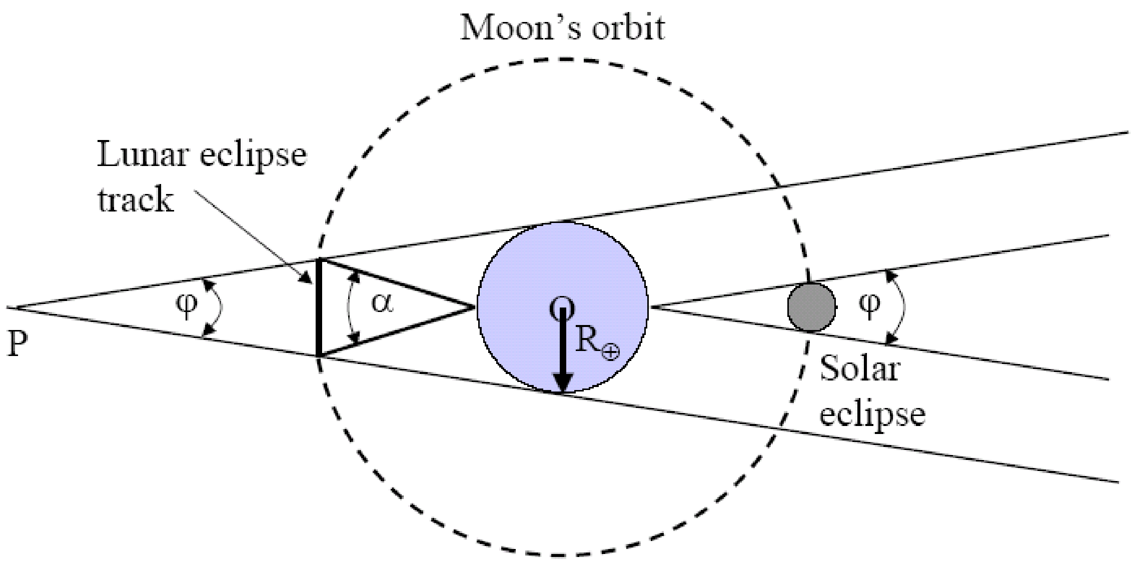

Figure 2: The geometry used to determine the angular width α of Earth's shadow at the Moon's orbital radius. The angle φ is the angular diameter of the Sun and Moon, taken in this analysis to be 0.5 degree.

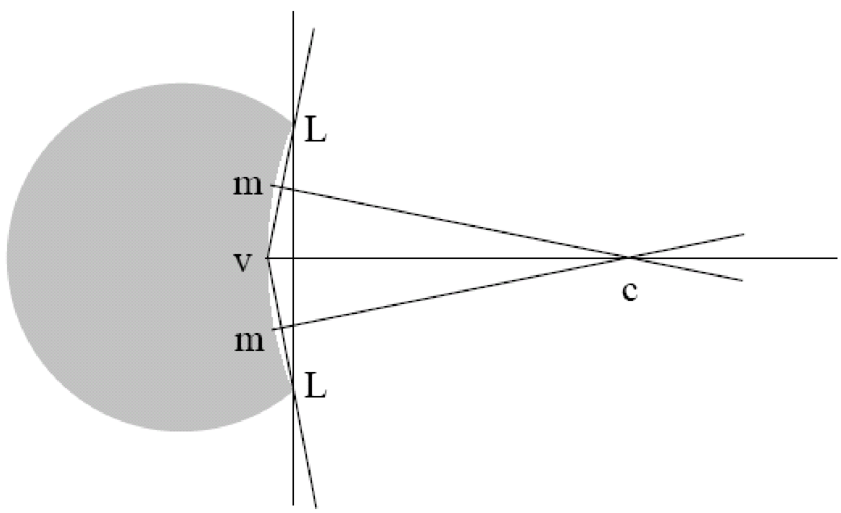

Figure 3. Method for the determination of the Earth's shadow radius at the Moon's orbit. The line LL crosses the Moon's limb at the same location as Earth's shadow. The perpendicular constructed at the mid-point of LL is extended to give point v. The perpendiculars at m are then constructed on lines vL and extended to meet at the center point for the Earth's shadow at c. The image scale is provided by assuming that the Moon has an angular diameter of 0.5 degrees.

The analysis illustrated in figure 3 builds upon the well known, indeed, classical geometrical result that a unique circle can always be constructed to pass through any three coplanar points, provided that the points do not all lie on the same line (this result is also expressed through the theorem which states there is a unique circumscribed circle to every triangle – Smith, 2000). As shown in figure 3, the center of the Earth's shadow is located at the interception point c of the perpendiculars constructed at the mid points m of segments vL. The point v is located by constructing a perpendicular at the mid-point of the line LL which is constructed to cross the Moon's limb at the intercept points of the Earth's shadow. Once the center point of the Earth's shadow c has been determined then the radius of the earth's shadow at the Moon's orbit is given by the distance cv (and cL). The angular width of the Earth's shadow α can then be determined from the diagram given the know angular size of the Moon ($\varphi = 0.5^o$)

Three lunar eclipse images were analyzed (via the method described in figure 3) in order to estimate the radius of the Earth's shadow at the Moon's orbit – the distance cv (see figure 4). The results are presented in table 1. Accordingly, I estimate that $\alpha = 1.34\ ^o$ = 0.0234 radians.

Time (Feb. 20, 2008) CST	Shadow width (degrees)	Shadow width (radians)
20:10	1.32	0.023
22:25	1.38	0.024
22:50	1.32	0.023

Table 1. Values derived for the angular diameter of the Earth's shadow at the Moon's orbit. The calculation assumes that the Moon has an angular diameter of 0.5 degrees.

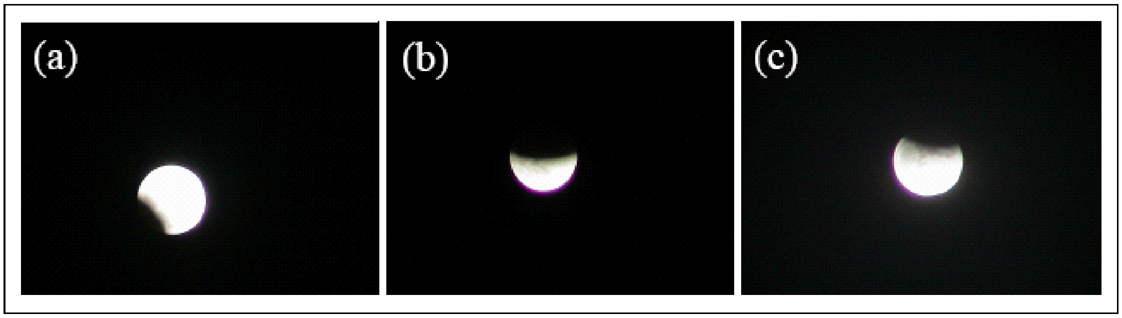

Figure 4: The three 'raw' partial eclipse images analyzed in order to estimate the radius of the Earth's shadow at the Moon's orbit. Each image was greatly enlarged in turn and analyzed according to the construction described in figure 3. All images were taken from Regina, Saskatchewan at the times indicated in column 1 of table 1 with a Sony 8 Handycam operating in single frame capture mode.

From equation (1) with $\varphi = 0.0087$ radians and $\alpha = 0.0234$ radians, my estimate for the Moon's orbital radius in units of Earth's radius comes out to be $D_☾ / R_\oplus = 61.58$. From the details given by Ptolemy it appears that Aristarchus didn't actually attempt to measure the diameter of the Earth's shadow at the Moon's orbit, rather he assumed it was twice as large as the Moon, corresponding to $\alpha = 1^o = 0.0175$ radians. This combination of numbers results in $D_☾ / R_\oplus = 75.67$. The great Hipparchus of Rhodes, working about a century after Aristarchus, estimated the Earth's shadow diameter at the Moon's orbit to be 8/3 times larger than that of the Moon (Kopal, 1970), corresponding to $\alpha = 1.33^o = 0.0232$ radians. With this value of α in equation (1) it is found that $D_☾ / R_\oplus = 61.97$. From Ptolemy's estimate for the shadow width of $\alpha = 1.30^o = 0.0227$ radians, we have $D_☾ / R_\oplus = 62.97$. Using the *Observers Handbook* we find that the Moon's mean orbital distance is 3.844×10^5 km and that the Earth's mean radius is 6371 km, giving $D_☾ / R_\oplus =$

60.34 [indicating that $\alpha = 1.37(8)^o = 0.0240(5)$ radians]. Not only did the ancient astronomers, Hipparchus and Ptolemy in particular, procure good estimates to the distance of the Moon, but also to my great surprise (being a mostly desk-bound writer and computer-screen reader) did I. My estimate of the relative distance to the Moon is about 2% too high. Combining this with an earlier estimate of 6294 km for the Earth's radius (Beech, 2005), the derived measure for the orbital radius of the Moon is ~387,600 km.

References

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