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A comment upon the ‘string’ ellipsograph

The December (2006) issue of the Bulletin carried a very enjoyable article by Dr. Allan Mills concerning ellipses and ellipsographs[1]. With respect to generating ellipses Dr. Mills describes the classical ‘string’ ellipsograph method (Mills, figure 5) in which a fixed length of string, having its two ends attached to pins at the focal points F_1 and F_2, is used to guide a pencil around an elliptical path. The reason for this letter, however, is to note that the ‘string’ method can be extended (at least in principle) to describe the so-called Cartesian ovals. As Dr. Mills[1] explains the generating equation for an ellipse can be written as $r_1 + r_2 = 2a$, where r_1 and r_2 give the distances to point P on the ellipse with respect to its two focal points F_1 and F_2, and where $2a$ is the major axis. For Cartesian ovals, however, the generating equation is $mr_1 + nr_2 = 1$, where m and n are constants. When $m = n$ an ellipse is generated, when $m = -n$ a hyperbola results, and in the special case $n = 1$ with m being any positive number greater than unity, the so called Limaçon of Pascal is produced[2]. Where all this relates back to the ‘string’ ellipsograph is through the experimentation of a young James Clerk Maxwell (1831 – 1879). Maxwell, famed for his electromagnetic theory of light, the theory of colours, the development of statistical mechanics and numerous other topics in mathematical physics[3], was just a teenager when he discovered the oval curve generation method and it was his first published paper[4].

With reference to figure 1, what the young Maxwell realized was that if one end of the constraining string is ‘attached’ to the pencil and looped around one of the focal points (F_1 in figure 1) then an oval would be produced with the corresponding generating equation of $2r_1 + r_2 = 1$ (i.e., the $m = 2$, $n = 1$ curve). In principle (but difficult in practice – I have tried!) additional numbers of loops can be added to the scheme. The next curve that might be drawn would be that with the generating equation $3r_1 + 2r_2 = 1$ – in this case there are two loops around F_1 and one loop around F_2.

The young Maxwell’s genius is truly remarkable, and, indeed, when his generalized method is described, one is immediately tempted to respond, “That’s so simple - why didn’t I think of it”. Perhaps just as remarkable is the fact that there is an actual application for Maxwell's method. For example, in the case where the generating equation is $3r_1 + 2r_2 = 1$ the resultant oval describes the edge profile required of an optical glass block, of refractive index 3/2, such that the light emitted from a lamp at one focus (situated outside of the glass block) is directed to the other focal point (situated inside of the glass block) via refraction. This is the refraction equivalent of the elliptical mirror mentioned by Dr. Mills. In principle, Maxwell's method can be extended to other materials for which the refractive index is m / n. The special optical properties of oval glass blocks was discussed by Christiaan Huygens (1629 - 1695) in his classical *Treatise on Light*, published in 1690, but Huygens used the idea of equal light travel times to determine the required profile, an approach that is now more commonly known under the name of Fermat's principle.

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Reference:

1. A. Mills. *Ellipses and ellipsographs*. Bull. Scientific Inst. Soc., No. 91 (2006), pp. 31 – 34.
2. J. D. Lawrence. *A catalog of special plane curves*. Dover Publications Inc. New York (1972). pp.155 -157.
3. Basil Mahon. *The man who changed everything – the life of James Clerk Maxwell*. John Wiley and Sons Ltd., Chichester (2003).
4. J. C. Maxwell. *On the description of oval curves, and those having a plurality of foci; with remarks by Professor Forbes*. Proc. Roy. Soc. Edinburgh, **2**, (1846).

Figure 1: The ‘string’ ellipsograph modified to generate a Limaçon of Pascal (a special case of the Cartesian ovals). The generating equation is $2r_1 + r_2 = 1$. The symbol A indicates the location of a string attachment point. The large solid arrow indicates the direction in which the pencil moves while keeping the string taut.