Feature Articles/
Articles de Fond

Atmospheric Height by Twilight's Glow

by Martin Beech, Campion College, University of Regina

A curse and a joy: these are the feelings brought on by twilight. To the astronomer who is sweeping for comets or searching out those final few Messier objects, the faint glow of dawn in the eastern sky is a curse, heralding the unstoppable on-rush of the Sun and an end to observing. The dawn twilight, in spite of its astronomy-stopping power, is still a beautiful sight to behold — it is indeed the muse of the love-struck poet. Likewise, but in contrast to the on-coming sunrise, the twilight following sunset is a joy for the astronomer since it heralds the onset of observing. Whether marking the end or the beginning of astronomical observing, the twilight phenomena should not be overlooked — there is science and discovery in those diaphanous glows — indeed, they tell us of the atmosphere.

Earth's atmosphere is a mere wisp of gas — a veneer-thin screen that protects us from the deadly ravages of space. Discussed since ancient antiquity, the atmosphere is the quintessential realm of change and material corruption. It is where weather takes place and where the elements of water, wind, and fire are mixed in pouring rain, swirling clouds, snow storms, and lightning: ever changeable, ever quixotic, ever the topic of discussion, and ever interesting. But, what are the physical characteristics of the atmosphere — how high is it and how does it vary with height? While visual observations of twilight phenomena can't provide an answer to the last of these questions, it can offer a guide to the first. Indeed, the answer was first determined in the 10th century by the remarkable Islamic scholar Abu Ali al-hasan ibn al-Hasan ibn al-Haytham (965 - 1039) who is better known through his Latinized name Alhazen. The geometry of the situation is quite straightforward and shown in Figure 1. The idea is that the minimum height of the atmosphere can be determined according to the limit set by solar illumination.

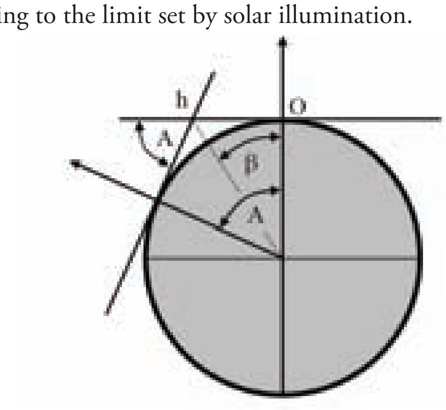

Figure 1 — The geometrical arrangement of Alhazen's twilight problem. The observer is located at point O and the zero-horizon illumination condition is satisfied once the Earth has spun through angle A. The height of the atmosphere is determined at the half angle $\beta = A/2$.

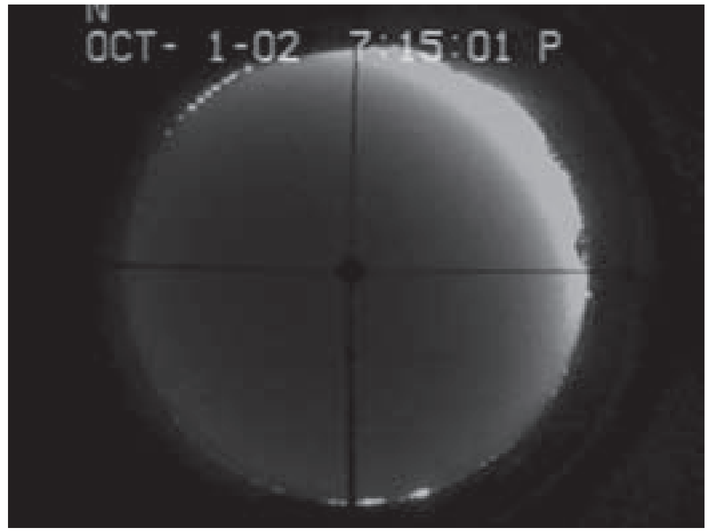

Figure 2 — One of all-sky images from the video sequence recorded on 2002 October 01 from Regina, Saskatchewan. The image is in a negative format and the twilight crescent is clearly visible in the western sky.

The trick is to gauge the angle A (the Sun's elevation below the horizon) at which the atmosphere is no longer appreciably illuminated by sunlight as viewed by the observer at O. Provided an estimate of angle A can be made, then the geometry indicates that the approximate height of the atmosphere is $h = R_E\ [\sec((\beta) - 1]$, where $\beta = A / 2$, and R_E is the Earth's radius. Alhazen realized that angle A could be estimated by measuring the duration ΔT of the twilight phenomenon — this is the time, for example, between the moment of sunset and the disappearance of the last skyglow on the horizon. Once ΔT has been measured, then the elevation of the Sun (below the horizon, of course) at the onset / or end moment of twilight can be determined. This result follows from the fact that the Sun appears to move through the sky at a rate of 15 degrees per hour. There is just one final twist, and this relates to the fact that in general the Sun does not rise and set along a vertical path with respect to the horizon — in general, the Sun's path will be at an angle $\alpha = 90 - \lambda$ degrees, where λ is the observer's latitude. Spherical trigonometry provides the final result that we want — namely: $\sin(A) = \sin(\alpha) \sin(\Delta T \times 15)$.

Inspired by a recent reading of Alhazen's works, it seemed only appropriate to try to repeat his twilight observations. Being far too old (or is that too wise?), to endure the wonders of a Saskatchewan sunset in the depths of winter (the time at which I was reading over Alhazen's works) I turned to a stock of canned video data collected with an all-sky video camera used primarily to monitor for fireball activity. An old videotape for 2002 October 1 seemed to fit the bill. Data collection on that night began prior to sunset and it was a glorious cloud-free night. The tape reveals that the sunset occurred at approximately 18:30 local time — perhaps it should be noted here that I had decided before starting the tape analysis to not check the actual sunset time until all the numbers were gathered in. The time of sunset is easily found, of course, with any planetarium program, and as it turned out, sunset was at 18:27 local time. A series of still images was obtained from the video sequence (Figure 2), and from these the height of the twilight crescent in the western sky was measured. In an attempt to determine a good estimate for the time of the last horizon glow, the projected radial distance from the mirror centre (where the image of the video camera itself is located) to the centre of the twilight arc was measured. The results of these measurements are shown in Figure 3. The gradual approach of the twilight arc to the horizon is clearly delineated from approximately 18:45 to 19:45, and, by fitting a smooth curve to the measured data points, the zero-horizon illumination time is predicted to be 19:55. The deduced twilight time interval was found to be ΔT = 75 minutes. With this deduced time interval the Sun's elevation below the horizon at the time of zero-horizon illumination is A = 11.9 degrees.

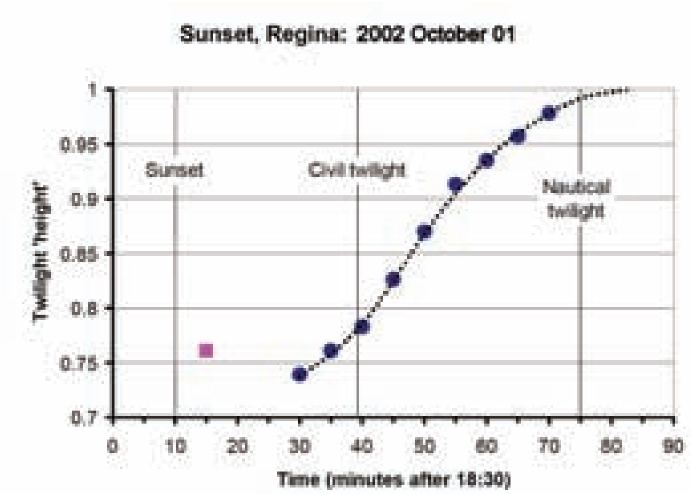

Figure 3 — Analysis of the projected distance from the centre of the mirror to the centre point of the twilight crescent (normalized with respect to the radius of the mirror). The dots correspond to the data points measured after 19:00 local time. The square corresponds to the first twilight arc measurement at 18:45. The vertical lines indicate the official times corresponding to sunset, the beginning of civil twilight, and the end of nautical twilight.

What we have actually determined in this analysis is an estimate for the end of nautical twilight defined as the time interval during which the Sun is between 6 and 12 degrees below the horizon. In practical terms, nautical twilight corresponds to the time during which the horizon becomes indistinguishable from the sky — this, of course, relates back to the pre-GPS days when maritime navigation was based upon being able to make sextant sightings. The interval prior to the onset of nautical twilight is civil twilight, which technically ends when the Sun is 6 degrees below the horizon. Civil twilight, as the name suggests, is inherently a legally set time limit beyond which many jurisdictions require, for example, the use of headlights on automobiles, and at which a daytime crime switches to becoming a nighttime crime. Astronomical twilight is deemed to begin at the end of nautical twilight and corresponds to solar elevations between 12 and 18 degrees. Nighttime is somewhat arbitrarily said to begin once magnitude +6.5 stars (the limit of human vision) become visible (under perfect viewing conditions) in the zenith.

Returning to Alhazen's realization that the duration of twilight can be used to estimate the height of the Earth's atmosphere, we have from the deduced horizon illumination condition that A = 11.9 degrees, and accordingly h = 34.7 km. This is clearly (from a modern perspective) a minimum height for the atmosphere, and with the video equipment we cannot go any further, since it does not detect star-like objects much fainter than magnitude -3. Nonetheless, with this minimum height estimate, the domain encompassing more than 80 percent of the atmosphere's mass has been enclosed, and over this altitude range the density will drop by a factor of order 100 of that recorded at ground level. In terms of the recognized atmospheric zones, an altitude of 35 km falls within the upper to middle region of the stratosphere, which also encompasses the upper region of the ozone layer.

To push the directly measurable height of the atmosphere higher than the twilight minimum just derived, one will have to investigate the heights of noctilucent clouds, meteors, and fireballs, and these will push the limit upwards by a factor of between 2 and 3 — indeed, comparable to that when the Sun's elevation angle A is some 18 degrees below the horizon.

Bibliography

Adam, John. A. (2009). *A Mathematical Nature Walk.* Princeton University Press

Freely, John (2009). *Aladdin's Lamp: How Greek science came to Europe through the Islamic world.* Alfred A. Knopf, New York.

Meinel, Aden and Marjorie (1991). *Sunsets, twilights and evening skies.* Cambridge University Press

Minnaert, M.G.J. (1993). *Light and color in the outdoors.* Springer-Verlag