The Measure of the Earth – a Saskatchewan diary

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Sunday June 20th, 2004. Regina. It is 6 a.m. I am up and awake and ready to measure the world – literally. Today is a day for old daydreams to be made true, and it is a day for retracing shadows of the ancient past.

When I first learnt as a young schoolboy how Eratosthenes had determined the size of the Earth by comparing the length of shadows cast at two different locations, I was enthralled. The concept was so very, very simple (as, indeed, are all leaps of genius once explained) and yet it was also so powerful. I reveled in the shear audacity of the idea - from shadow lengths to the circumference of the Earth. Here was the stamp of human imagination, and here was the 'measure in all things' as espoused by Horace.

It is 6:20 a.m. The weather, as per usual in Saskatchewan, is not cooperating; the sky is a uniform gray. The forecast, however, is for intermittent clear spells with perhaps an occasional shower. I have set up the first of the sundials in our back garden and Georgette [my wife] *will make one set of observations from there.*

The clock has just rounded 7 a.m. and I am traveling north out of Regina, heading for Star City, on highway number 6 [see figure 1]. *I will set up the second sundial there, and a better-named location from which to make my measurements I could not hope to find. The skies are still overcast and an intermittent light rain is falling.*

There is something wonderfully compelling about the astronomy practiced by the ancient Greek philosophers. Their universe was simple, elegant, compact and completely known – indeed, it was almost everything (elegant aside, perhaps) that our modern day universe isn't. In keeping with his time the calculation performed by Eratosthenes was new, bold and highly imaginative. He began from the principle that the Earth was a sphere and from there through the application of elementary geometry, and three actual measurements, he derived the girth of our home world. Incredible!

Eratosthenes was born circa 275 BC in Cyrene, a then Greek City on what is now the North African coast of Libya. In later life he moved to Alexandria and distinguished himself as a librarian, philosopher, poet and athlete[1]. No fragments of Eratosthenes original writings concerning the measure of the Earth have survived to the modern era, but an account of his procedure is given by Clemodes in his *On the Elementary Theory of the Heavenly Bodies*. Clemodes explains that Eratosthenes method made specific use of observations relating to the altitude of the Sun[2]. In particular Eratosthenes noted that in Syene (now the city of Aswan) the Sun was directly overhead at noon on the day of the summer solstice, while in Alexandria, some 5000 stades to the north, the Sun was 1/50 of a circle away from the zenith[3]. With these pieces of information, admittedly none of which, as far as we know, were actually measured by Eratosthenes, he was able to determine that the Earth had a circumference of 252,000 stades[4].

It is 9:34 a.m. I have just arrived in Watson, over half way to Star City. The weather is beginning to improve. I can at last see clear breaks of blue sky.

It is 10:33 a.m. and I have stopped for coffee at Melfort. I am now just a few tens of kilometers from Star City, and the wind is howling. Dirty-gray and ominous heaps of cumulus clouds cover most of the sky - but clear breaks are visible. During the past hour the Sun has occasionally blazed out from between clouds, but typically for just a few tens of minutes. The weather isn't perfect, but we should be able to work around it. Cell phone contact with Regina indicates that it is clear and sunny there.

It is 11:45 a.m., and I am standing by my car. It is parked on a diagonal in an attempt to act as a windbreak. The sundial has been set in place and its base has been leveled [see figure 2]. *I am situated some 10-km north of Star City, at a roadside turnoff. It is a desolate spot; the wind is gusting over the open fields and the verge side grasses are rippling and bending wildly in its path. I catch my breath – now for the first measurement. I call through to Regina and the first simultaneous twin marking of sundial shadow lengths is achieved at 11:47 a.m.*

It is 11:55 a.m. a brooding mass of dark cloud has moved overhead. A heavy rain has begun to fall and the sundial has just been blown over by a tremendous blast of wind. I rush to get the sundial safe inside the car – I don't want its wooden frame to get wet. Oh, well, no one said that measuring the Earth was going to be easy. We have one data set all ready, and moving the sundial is not a great problem since it is shadow lengths that we are measuring, not relative shadow motion. I have moved the car to place it as a better windbreak.

It is 12:22 p.m. The rain has cleared away, the wind has dropped to a whisper, and we have just successfully completed another simultaneous shadow length measurement. The air is pungent with the smell of rain washed earth. It is as if a deep and refreshing breath has been drawn-in by the land with the exhalation held back, for just a few short minutes, in order to stay the invigorating enjoyment of the moment. I can't help but feel that this must be something like the astronomy of the ancients: out on the land, feeling and sensing the solid Earth beneath ones feet.

It is 2:00 p.m. A steady rain has begun to fall and I am now ready to head for home. We have gathered simultaneous shadow length measurements at 1:00, 1: 12 and 1:43 p.m. A good haul of data points – I hope.

It is 6:26 p.m. Home! The drive back has been long and bothersome. I am tired. A check of the odometer reveals a distance of 287.8 km between the Regina sundial station and that at Star City [5]. I will rest for an hour and then turn to the numbers.

Table 1 is a summary of the measured shadow lengths and resultant Sun altitudes from Regina and Star City. For the purpose of measuring the Earth, it is the difference in Sun altitudes, α, that is important.

Time (CST)	Shadow length (mm): Regina	Altitude (deg.)	Shadow length (mm): Star City	Altitude (deg.)	α (deg.)
11:47	92.5	59.81	103.0	57.06	2.75
12:22	85.0	61.87	94.0	59.41	2.46
13:00	81.5	62.86	91.0	60.22	2.64
13:12	81.5	62.86	92.0	59.95	2.91
13:43	86.5	61.45	95.5	59.01	2.44

Table 1. Sundial shadow lengths and Sun altitudes from Regina and Star City. Each sundial has a gnomon of length 15.9-cm, and the tangent of the Sun's altitude is simply the ratio of the gnomon and shadow lengths. The 6^{th} column shows the difference in the Sun's altitude as measured at the two locations; ideally the difference should be a constant.

The essential geometry of the measurements made at Regina and Star City is shown in figure 3, and just as Eratosthenes would have calculated it, the circumference of the Earth is given by the formula

$$\text{Circumference} = 2\,\pi\,R_{\oplus} = (360\,/\,\alpha) \times D(\text{km})$$

Where $R_{\oplus}$ is the Earth's radius and D is the distance between Regina and Star City in kilometers. From my car's odometer I have D(km) = 287.8 ± 0.2. From column six in table 1, I also have α (deg.) = 2.62 ± 0.12. These numbers combine to give an estimate of the Earth's radius of $R_{\oplus}$ = 6294-km, with a formal uncertainty[6] of 4.6%. The RASC Observer's Handbook gives the Earth's mean radius as 6371-km.

10:25 p.m. Well, not so bad a days outing. Have measured the size of the Earth to within a few percent of its 'correct' value, and completed, finally, one of those adventures planed many long years ago.

Notes:

1) Eratosthenes was also a well-known and celebrated mathematician. Indeed Archimedes specifically records that Eratosthenes described an instrument capable of duplicating a cube, and that he also developed a set of rules for generating prime number tables - via the, so-called, sieve of Eratosthenes. In later life Eratosthenes lost his eyesight and refusing to live if he could not read he committed suicide in 194 B.C.
2) See James Evans, *The History and Practice of Ancient Astronomy* (OUP, Oxford, 1998, pp.63-66). While Eratosthenes calculation was based upon measurements supposedly gathered at noon on the day of the Summer Solstice, the calculation can be made on any day of the year. In addition to using the Sun to determine the latitude

difference between two observing locations, one can also use star altitudes at their lower culmination. Posidonius circa 100 B.C., for example, used observations of the bright star Canopus, as seen by observers at Rhodes and Alexandria, to deduce that the Earth's circumference was some 240,000 stades.

3) The 1/50th of a circle measurement is equivalent to 360 / 50 = 7.2 degrees.
4) Stades are a well-known problematic unit in the sense that they have no standard. Zdenek Kopal in his *Widening Horizons* (Taplinger Publishing, New York, 1970, p. 18) argues that 1 Stade is equivalent to about 1/6th of a kilometer. If this conversion is correct then the radius of the Earth as deduced by Eratosthenes is of order 6317 km.
5) This is the northing distance. I have taken off the easterly distance from Melfort to the Star City turnoff. Technically the observations should be made from locations that are on the same meridian, but a small offset to the east for the second station is not of major concern for the calculation presented here.
6) The uncertainty in the circumference is $\Delta C / C = \Delta D / D + \Delta\alpha / \alpha$, where we estimate $\Delta D = 0.2$ km, and where $\Delta\alpha = 0.12$ from the values presented in table 1.

Figures

Figure 1: Schematic map of Saskatchewan showing the sundial locations at Regina and Star City.

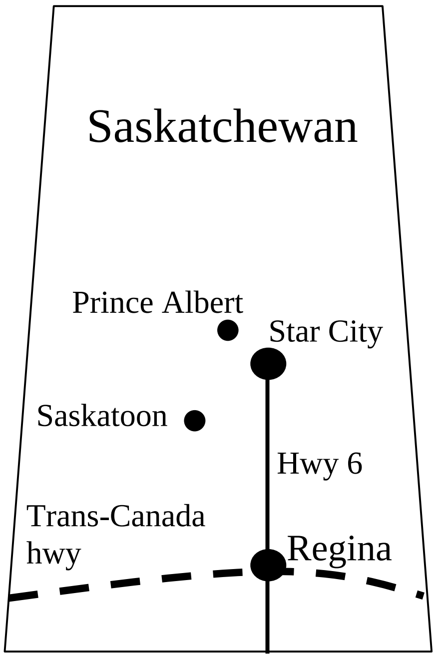

Figure 2: The portable sundial as setup near Star City.

Figure 3: The geometry for finding the size of the Earth from shadow length measurements on sundials at two locations a known distance D apart. The angle $\alpha = \beta - \gamma$ corresponds to the difference in the latitudes of the two observing locations; R and SC correspond to the locations of Regina and Star City where the angles γ and β are measured respectively.